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MASS DEPENDENT α_S EVOLUTION
and
THE LIGHT GLUINO EXISTENCE

D.V. Shirkov [1]

Bogoliubov Theoretical Lab., JINR, Dubna, Russia

S.V. Mikhailov [2]

Bogoliubov Theoretical Lab., JINR, Dubna, Russia
and
Sektion Physik, Bielefeld Uni., Bielefeld, Germany

Abstract

There is an intriguing discrepancy between $\bar{\alpha}_s(M_Z)$ values measured directly at the CERN Z_0-factory and low-energy (at few GeV) measurements transformed to $Q = M_{Z_0}$ by a massless QCD $\bar{\alpha}_s(Q)$ evolution relation. There exists an attempt to reconcile this discrepancy by introducing a light gluino $\tilde{g}$ in the MSSM.

We study in detail the influence of heavy thresholds on $\bar{\alpha}_s(Q)$ evolution. First, we consruct the "exact" explicit solution to the mass-dependent two-loop RG equation for the running $\bar{\alpha}_s(Q)$. This solution describes heavy thresholds smoothly. Second, we use this solution to recalculate anew $\bar{\alpha}_s(M_Z)$ values corresponding to "low-energy" input data.

Our analysis demonstrates that using *mass-dependent RG procedure* generally produces corrections of two types: Asymptotic correction due to effective shift of threshold position; Local threshold correction only for the case when input experiment lies in the close vicinity of heavy particle threshold: $Q_{expt} \simeq M_h$

Both effects result in the effective shift of the $\bar{\alpha}_s(M_Z)$ values of the order of 10^{-3}. However, the second one could be enhanced when the gluino mass is close to a heavy quark mass. For such a case the sum effect could be important for the discussion of the light gluino existence as it further changes the $\tilde{g}$ mass.

[1]e-mail: shirkovd@th-head.jinr.dubna.su
[2]e-mail: mikhs@theor.jinrc.dubna.su

1   Introduction

Discrepancy for $\bar{\alpha}_s(M_Z)$ value. Recent experiments at CERN and DESY provide the possibility of a rather accurate (up to few per cent) comparison of data with the QCD predictions for strong running coupling $\bar{\alpha}_s(Q)$ evolution [1]. The detailed theoretical analysis results [2] in the conclusion that there exists a slight discrepancy between two groups of experiments, on the one hand, and "standard (i.e., treated as a part of SM) QCD", on the other.

So we are confronted with a new "Bermuda triangle". This time, unlike the first mismatch [3] connected with GUT speculation at distances of order 10^{-30}cm, the new puzzle (if it really exists) concerns physics at the current frontier of momentum transfer.

The nature of this new discrepancy admits speculations on the existence of superpartners with masses lighter than M_{Z_0} [4],[5],[2],[6] that is on a drastic decrease of the SUSY scale .

Light superpartners, generally, reveal themselves in two ways: they slow down [4], [5] the running of $\bar{\alpha}_s(Q)$ and, besides, they modify the analysis of experimental data (see, e.g., Refs.[2], [7] and [8]).

Our program. The aim of this paper is two-fold.

The first is to demonstrate the advantage of a mass-dependent renormalization group (RG) technique for analysing coupling constant evolution. This technique is based upon the original Bogoliubov formulation of the RG, the formulation that is not tightly related to short-distance behaviour, massless approximation and UV divergencies. It uses as input *mass-dependent perturbative calculation* and takes automatically into account threshold effects. As this approach usually corresponds to MOM-schemes it can be simply connected with experimental conditions. It is appropriate to modern α_s measurements at few GeV in decays and DIS where heavy particle thresholds play a noticeable role. For our discussion we use a mass-dependent – "massive" for short – generalization (see Eq.(2.2) below) of the well-known two-loop UV massless RG solution for the running coupling

$$\bar{\alpha}^{(2)}(Q) = \frac{\alpha}{1 + \alpha\beta_1\ell + \alpha\,(\beta_2/\beta_1)\ln(1 + \alpha\beta_1\ell)} \;; \quad \ell = \ln(\frac{Q^2}{\mu^2}) \;. \tag{1.1}$$

Our second goal consists of discussing of the above-mentioned discrepancy between the values of $\bar{\alpha}_s(M_Z)$ on the basis of mass-dependent $\bar{\alpha}_s(Q)$ evolution.

Results. To this end we consider details of this evolution in the threshold vicinity and discover effect coming from two sources:

a. Asymptotic correction due to the effective threshold shift ,*e.g.*, $Q_{thr} = 2m \to 2.30m$.

b. Local threshold correction only for the case when input experiment momentum lies in the close vicinity of heavy quark (or gluino) threshold: $q_{expt} \simeq M_h$.

Each effect results in effective shift of the $\bar{\alpha}_s(M_Z)$ values of order of 0.001. However, the second correction could be enhanced when gluino mass happens to be close to the heavy quark one[3]. Here the summary effect could be of order 0.005 that is important for the discussion of a light gluino hypothesis.

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[3] One could say that in this case gluino plays a role of a "magnifying glass" for threshold effect.

2 Mass-dependent $\bar{\alpha}_s(Q)$ evolution

Analytic mass-dependent $\bar{\alpha}_s(Q)$ evolution on the two-loop level. An exact solution to the one-loop massive RG equation for the running coupling $\bar{\alpha}_s(Q)$ is known from the mid-fifties [9],[10]. If one starts with the one-loop mass-dependent perturbative input, then the radiative correction, after applying RG machinery, goes into the denominator in the same manner as for the massless case:

$$\bar{\alpha}_{pert}^{(1)}(Q^2,m^2,\mu^2) = \alpha - \alpha^2 \cdot U_1(Q^2,m^2,\mu^2) \quad \rightarrow \quad \bar{\alpha}_{RG}^{(1)}(Q) = \frac{\alpha}{1+\alpha U_1} \; . \tag{2.1}$$

On the two-loop level starting with

$$\bar{\alpha}_{pert}^{(2)}(Q,...) = \alpha - \alpha^2 \cdot U_1 + \alpha^3 \cdot \left(U_1^2 - U_2\right) + ...$$

one can also obtain [11] an analytic solution

$$\bar{\alpha}_{RG}^{(2)}(Q,m) = \frac{\alpha}{1 + \alpha U_1 + \alpha \left(U_2/U_1\right)\ln(1+\alpha U_1)}. \tag{2.2}$$

Here the l-loop contribution U_l in a MOM-scheme has a simple functional structure [11]

$$U_l(Q^2,\ldots) = I_l(Q^2/m^2) - I_l(\mu^2/m^2) \tag{2.3}$$

and μ^2 is the Euclidean subtraction point.

The RG solution (2.2) possesses several remarkable properties. First, it depends only on the loop-expansion coefficients U_1 and U_2 just in the form they appear in the perturbative input. Second, in the pure massless case with $U_l(Q^2,...) = \beta_l \ln(Q^2/\mu^2)$ Eq.(2.2) precisely corresponds to the well-known expression (1.1). Third, being used in QCD, it smoothly interpolates across the heavy quark thresholds between corresponding massless expressions with different quark numbers n_f. Finally, expression (2.2) can be considered as self-consistent on the two-loop level, as the error of approximation involved is of the next-to-next-to-leading, i.e., the three-loop, order (as it was shown explicitly in Ref.[12]). In this sense we shall refer to it as to an *exact two-loop massive solution.*

One-loop qualitative discussion. Result of U_1 perturbative QCD calculation depends on the renormalization scheme, but *not* on the gauge when incoming fermions are massless (for the general case see [13]).

The one-loop massive coefficient U_1 (and the corresponding β-function) was calculated first in [14] (see also [15]) on the basis of the ghost-gluon vertex (U_1^{ggA}), then a different expression ($U_1^{(AAA)}$) was obtained [16] with the triple-gluon vertex. The $\bar{\alpha}_s(Q)$ evolution based on the quark-gluon vertex with a massive incoming quarks was considered in [13]). The need of a concrete $\bar{\alpha}_s(Q)$ is determined by the tree structure of diagrams for a given process. For example, for non-singlet quark distribution in DIS processes the renormalized triple-gluon vertex does not occur in the leading *log* diagrams; nor in quarkonium decays. So, we shall use only $\bar{\alpha}_s(Q)^{ggA}$ in the following.

In the QCD case functions U_l consist of a sum of massless and massive terms

$$U_l(Q^2, \mu^2, ..m_h^2, ..) = \beta_l \cdot \ln(\frac{Q^2}{\mu^2}) - \sum_h \Delta\beta_l^h \cdot H_l^h(Q^2, \mu^2, m_h^2), \tag{2.4}$$

$$H_l^h(Q^2, \mu^2, m_h^2) = I_l^h(\frac{Q^2}{m_h^2}) - I_l^h(\frac{\mu^2}{m_h^2})$$

with $12\pi\beta_1 = 11C_A - 2n_f$, $48\pi^2\beta_2 = 34C_A^2 - 38n_f$, n_f being the number of light quarks and the sum extended over all kinds h of heavy particles. In particular, on the one-loop level one has for quark contribution $C_A = 3$, $\Delta\beta_1^q = 1/6\pi$ and

$$I_1^q(z) = 2\sqrt{1 + \frac{4}{z}}\left(1 - \frac{2}{z}\right)\ln\left(\sqrt{1 + \frac{z}{4}} + \sqrt{\frac{z}{4}}\right) + \frac{4}{z} - \frac{5}{3} \quad ; \tag{2.5}$$

$$I_1^q(z \to \infty) \to \ln z - \frac{5}{3} + O(1/z) \quad ; \quad I_1^q(z \to 0) \to \frac{z}{5} \quad . \tag{2.6}$$

The constant $C_1 = 5/3$ in the asymptotic form (2.6) can be considered as *effectively shifting the threshold position* in the asymptotic logarithmic contribution

$$I_1^q(Q^2/m^2 \to \infty) \to \ln(Q^2/M^2) \quad ; \quad m \to M = m\exp(5/6) = 2.301m. \tag{2.7}$$

One can arrive at an exact solution for $\bar{\alpha} = \bar{\alpha}_s$ by substituting expressions (2.5) and (2.4) into Eq.(2.1). In the MSSM gluino case one must appropriately change the C_A factor (due to the Majorana spinor nature) in the coefficient $\Delta\beta_1^q$ in (2.4).

Note also that the heavy particles' contribution, as it stands under the sum on the r.h.s. of Eq.(2.4), does not satisfy a decoupling condition at small Q^2. It rather corresponds to the case with $\mu \simeq m_h = m_c, m_b, m_{\tilde{g}} << M_Z$. This perfectly fits our physical situation with $\alpha_s(\mu \simeq \text{few GeV})$ used as input and $\bar{\alpha}_s(M_Z)$ being the output.

However, in some cases, (e.g., in discussion of the GUT consistency problem based upon all three SM couplings $\alpha_i(M_Z)$ experimental values) one should satisfy the decoupling theorem with respect to all possible particles heavier than given scale (M_{Z_0}) and omit the $I_l^h(\mu^2/m_h^2)$ terms in the sum.

Simple logarithmic approximations. As the exact analytic form of the fermion one-loop contribution Eq.(2.5) is rather cumbersome, there was proposed (see [14]) a simple approximation for I_1^q

$$\ln(1 + \frac{z}{5}) = \ln\left(1 + \frac{Q^2}{5m^2}\right) \tag{2.8}$$

which has reasonable limits at small and large Q^2 (compare with Eq.(2.6)) and for intermediate Q^2 is "accurate within a few per cent". We see now that a simple modification of this expression :

$$\tilde{I}_1^q(z) \approx \ln(1 + z/5.3) \tag{2.9}$$

restores the exact asymptotic behaviour of $I_1^q(z)$ and leads to absolute accuracy of the 10^{-2} order (for $z \geq 1$) that is quite enough for our discussion.

Two-loop massive contribution to $\bar\alpha_s(Q)$. To avoid a complicated issue of the gauge dependence, we shall use (see, e.g., Refs. [17], [18]) the transverse (i.e., Landau) gauge. For our discussion we need an *explicit two-loop mass-dependent contribution* to the QCD running coupling. This important calculation for fermion contribution has been performed by Yoshino and Hagiwara [19]. Unhappily, these authors have published their original result neither for U_2 nor for contribution to the β-function. The only published expression is the rational parameterization for the next-to-leading order coefficient of the β-function $\beta_{ggA}^{(2)}$ in the MOM scheme at the symmetrical Euclidean point for the ghost-gluon vertex in the Landau gauge:

$$(4\pi)^2\beta_H^{(2)} \approx -\frac{38}{3}\sum_h B_2(\frac{Q^2}{m_h^2}) \; ; \quad B_2(z) = \frac{0.26995z^2 - 0.45577z}{0.26995z^2 + 2.1742z + 1} \; .$$

We restore the corresponding approximate expression for $I_2^q(z)$ by elementary integration :

$$
\begin{aligned}
I_2^h(z) &= A\ln(1 + z/z_1) + (1 - A)\ln(1 + z/z_2) \; ; & (2.10)\\
A &= 1.310 \; ; \quad z_1 = 7.579 \; ; \quad z_2 = 0.4897 \; .
\end{aligned}
$$

It is easily seen now, that the "effect of the threshold shift" increases at the two-loop level

$$I_2^q(z \to \infty) = \ln z - C_2 \; ; \quad C_2 = [A\ln(z_1) + (1 - A)\ln(z_2)] \approx 2.872.$$

as compared with the value $C_1 = 5/3$ in (2.6).

Using now expression (2.10) in (2.4) with $(4\pi)^2\Delta\beta_2^q = 38/3$ (for the gluino contribution $(4\pi)^2\Delta\beta_2^{\tilde g} = 48$ [4],[5]) and then in Eq.(2.2) we obtain an explicit mass-dependent two-loop evolution law for the QCD running coupling $\bar\alpha_s^{(MOM)}$.

3 Analysis of threshold effects in $\bar\alpha_s(Q)$ evolution

Thresholds in massless MS schemes. The usual way to account for heavy threshold effects in massless $\bar\alpha_s(Q)$ evolution consists in imposing the continuity relations

$$\alpha_s[n - 1, M_n] = \alpha_s[n, M_n] \tag{3.1}$$

with M_n being the point of "switching on" the n-th quark, i.e., some effective threshold value[4]. This procedure leads to a "spline-type" approximation for massless $\bar\alpha_s(Q)$.

Quite often for the matching point one takes (see, e.g., widely cited Marciano paper[20]) the n-th particle mass value $M_n = m_n$. "To be more accurate" sometimes (see,*e.g.*, [2]) one uses the threshold position $M_n = 2m_n$. However, as it follows from Eqs.(2.1), (2.7), the optimum value of the adjusting parameter in the relation $M_n = k\cdot m_n$ for fermion polarization

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[4]It should be noted, that this recipe hasn't rigorous substantiation in the field theory and is devoted to imitate some final results of the mass-dependent calculation.

loop (that corresponds to $\beta_{ggA}^{(1)}$) is $k_{opt}^{(1)} = 2.301$[5] and on the two-loop level even to $k^{(2)} \approx 4.404$. The maximum deviation of the spline approximation $\theta(Q - M_h) \ln(Q^2/M_h^2)$ for I_1^h from the exact continuous mass-dependent expression (2.5) lies in the threshold vicinity, being of order of unity.

However, in real QCD we face with a more complicated situation: thresholds of heavy quarks and, possibly, of a light gluino are located rather close to each other. This leads to their mutual influence that could amplify massive correction near the threshold region.

Different schemes at one loop level. For qualitative illustrative discussion consider first the one-loop case with a heavy mass m_n. Here it is convenient to represent the QCD running coupling taken in some i-th subtraction scheme in the form:

a) in the massless case

$$\frac{1}{\bar{\alpha}_i(Q, m = 0)} = \frac{1}{\bar{\alpha}_{i,0}(Q)} = \frac{1}{\alpha_\mu^i} + \beta_{n^*}\ell + c_{i,n^*} \; ; \quad \ell = \ln(\frac{Q^2}{\mu^2}) \tag{3.2}$$

with

$$n^* = n - 1 \;\; \text{at} \;\; Q \leq m_n \; ; \;\; = n \;\; \text{at} \;\; Q \geq m_n \; ; \;\; \text{and} \;\; \beta_n = \beta_{n-1} - \Delta\beta_1^n;$$

b) in the massive case

$$\frac{1}{\bar{\alpha}_i(Q, m)} = \frac{1}{\alpha_\mu^i} + \beta_{n-1}\ell - \Delta\beta_1^n \cdot \left[I_1^n\left(\frac{Q^2}{m_n^2}\right) + C_i\left(\frac{\mu}{m_n}\right) \right] \tag{3.3}$$

where are c_i, C_i some scheme dependent constants.

In particular, for the widely accepted $\overline{\text{MS}}$ scheme we have two versions:
The massless

$$\frac{1}{\bar{\alpha}_{\overline{\text{MS}},o}(Q)} = \frac{1}{\alpha_{\mu,n^*}} + \beta_{n^*}\left(\ell - \frac{5}{3}\right) \tag{3.4}$$

and the massive one

$$\frac{1}{\bar{\alpha}_{\overline{\text{MS}}}(Q, m)} = \frac{1}{\alpha_{\mu,n}} + \beta_{n-1}\left(\ell - \frac{5}{3}\right) - \Delta\beta_1^n \cdot \left\{ I_1^n\left(\frac{Q^2}{m_n^2}\right) - \ln\frac{\mu^2}{m_n^2} \right\} \; , \tag{3.5}$$

which evidently relate to each other in the limit $m_n \to 0$ with appropriate change

$$\frac{1}{\alpha_{\mu,n-1}} = \frac{1}{\alpha_{\mu,n}} - \Delta\beta_1^n\left[\ln\frac{Q^2}{\mu^2} - \frac{5}{3} \right] \; . \tag{3.6}$$

Note here that for practical application sometimes one naively uses the massless Eq.(3.4) in the form

$$\frac{1}{\bar{\alpha}_{\overline{\text{MS}}}(Q)} = \frac{1}{\alpha_{\overline{\text{MS}},n^*}(\mu)} + \beta_{n^*}\ell \tag{3.7}$$

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[5]It is rather curious that in Ref. [21] this value has been found empirically!

with continuity or *matching* condition at some $Q = M_n \simeq m_n$ that relates coupling constants

$$\frac{1}{\alpha_{\mu,n-1}} = \frac{1}{\alpha_{\mu,n}} - \Delta\beta_1^n \left[\ln \frac{M_n^2}{\mu^2} - \frac{5}{3} \right] . \tag{3.8}$$

For further discussion we have to consider the $\bar{\alpha}_s(Q)$ evolution from some low-energy input scale $\mu = q_{in} \simeq m_{c,b,...}$ up to $Q = M_{Z_0}$ value. Usually to this end people use massless Eq.(3.4) which we write down in the spline form

$$\frac{1}{\bar{\alpha}_{\overline{MS}}(Q)} = \frac{1}{\alpha_{\overline{MS}}(\mu)} + \beta_{n-1}\ell - \Delta\beta_1^n \cdot \theta(Q - M_n) \ln \frac{Q^2}{M_n^2} \tag{3.9}$$

that corresponds to Eq.(3.7).

We shall also use the exact smooth massive expression (3.3) in the MOM scheme

$$\frac{1}{\bar{\alpha}_{MOM}(Q,m)} = \frac{1}{\alpha_{MOM}(\mu)} + \beta_{n-1}\ell - \Delta\beta_1^n \left\{ I_1^n \left(\frac{Q^2}{m_n^2} \right) - I_1 \left(\frac{\mu^2}{m_n^2} \right) \right\} . \tag{3.10}$$

Qualitative comparison of evolution in different schemes. The important point one has to fix before the comparison of the exact smooth Eq.(3.10) and massless spline-type Eq.(3.9) evolution laws is the relation to experimentally measured quantities.

Being a bit simple-minded we suppose that the experimental input value $\alpha_s(q_{in})$ can be equally treated as $\alpha_{\overline{MS}}(q_{in})$ in Eq.(3.9) or as $\alpha_{MOM}(\mu = q_{in})$ in Eq.(3.10). Under this convention we have

$$\frac{1}{\alpha_{MOM}(M_Z)} - \frac{1}{\alpha_{\overline{MS}}(M_Z)} = \sum_h \Delta\beta_1^h \cdot \left\{ I_1^h \left(\frac{q_{in}^2}{m_h^2} \right) + \frac{5}{3} - \ln \frac{M_h^2}{m_h^2} \right\} . \tag{3.11}$$

As it follows from this result generally there are two corrections to the $(\bar{\alpha}_s(M_Z))^{-1}$ value. The first, "asymptotic", one contains the difference $\frac{5}{3} - \ln k^2$ with $1 \le k = M_h/m_h \le k_{opt}^{(1)}$ and, at least partially, sometimes is taken into account by the change of the matching point. However, the second, really "threshold correction", in the current literature is not taken into account at all. This contribution

$$\Delta_{thr} \frac{1}{\bar{\alpha}_s(M_Z)} = \sum_h \Delta\beta_1^h \cdot I_1^h \left(\frac{q_{in}^2}{m_h^2} \right) \tag{3.12}$$

is important for the case when

input momentum transfer is close to some heavy particle threshold.

Postponing the numerical analysis for the last Section, we make here few comments:

1. As $I_1(z \simeq 1) \simeq 1$ the threshold correction Eq.(3.12) for $m_c < q_{in} < m_b$ could be of the order of $0.05 - 0.10$ which results in the relative effect $(\Delta_{thr}\bar{\alpha}_s(M_Z)) / \bar{\alpha}_s(M_Z) \simeq 10^{-2}$.

2. The abovementioned asymptotic effect takes maximum value for $k = 1$ and for each of the c and b thresholds produces the same effect $\Delta_{as}\bar{\alpha}_s(M_Z)/\bar{\alpha}_s(M_Z) \equiv (1 \div 2) \cdot 10^{-2}$.

3. Second-loop effects also could be quantitatively estimated on the base of explicit expression Eq.(2.10). This leads to the effective enhancement of one-loop effects by 20 - 30 per cent.

$\overline{\text{MS}} \to MOM \to \overline{\text{MS}}$ *transitions*. The other way to take the threshold into account is to treat the experimental input value $\alpha_s(q_{in})$ as $\alpha_{\mu,n-1}$ in Eq.(3.4) or (3.5). The procedure one should use for this situation contains three steps:

(i) transformation of $\bar{\alpha}_s(Q)$ from $\overline{\text{MS}}$ to a MOM-scheme at the low energy scale q_{in};

(ii) evolution of $\alpha_s^{MOM}(q_{in})$ to $\alpha_s^{MOM}(M_Z)$ by Eq.(2.2) that describes smoothly the gluino and heavy quarks thresholds ;

(iii) return transformation from $\alpha_s^{MOM}(M_Z)$ to $\alpha_s^{\overline{\text{MS}}}(M_Z)$

that is:

$$\begin{aligned}
\overline{\text{MS}} &\to MOM \quad \text{at} \quad q_{in} \simeq m_{c,b} \ ; \\
\alpha_s^{MOM}(q_{in}) &\to \ \alpha_s^{MOM}(M_Z) \quad \text{via Eq.(2.2)}; \\
MOM &\to \overline{\text{MS}} \ \text{at} \ Q = M_Z.
\end{aligned} \tag{3.13}$$

To connect our equations with data for $\mu = q_{in}$ and $Q = M_Z$ (which are extracted from experiment by using $\overline{\text{MS}}$ –scheme) we need relations between $\overline{\text{MS}}$ (see (3.4) or (3.5)) and our version of MOM - scheme.

Here is the relation

$$\frac{1}{\alpha_{MOM}} = \frac{1}{\alpha_{\overline{\text{MS}}}} - A_1 - \alpha_{\overline{\text{MS}}} \cdot A_2; \qquad \frac{1}{\alpha_{\overline{\text{MS}}}} = \frac{1}{\alpha_{MOM}} + A_1 + \alpha_{MOM} \cdot A_2 \tag{3.14}$$

where A_1 was represented, *e.g.* in [19], with the constant $D_1 = (5/3)\beta_1 - (11/12)C_A$,

$$A_l(\mu^2/m_h^2, ..) = \frac{1}{(4\pi)^l} \left(\sum_h \Delta\beta_l^h \cdot \left(I_l^h(\mu^2/m_h^2) - \ln(\mu^2/m_h^2) \right) + D_l \right), \tag{3.15}$$

and the coefficient A_2 may be extracted from (2.10) with D_2 being some constant dependent on the MOM-scheme chosen. When using Eqs. (2.10) at the high energy (HE) scale $Q = M_Z$ we have to extend the sum in (3.15) to the c, b- quarks and perhaps gluino. In the low energy (LE) case, quark contributions appear in the sum as the corresponding thresholds M_h are passed at the scale $\mu = q_{in}$. The final result of the three successive steps (3.13) is given by formula (2.2) with the new U_l (which differ now from the expression (2.4)) :

$$U_l^{New}(Q^2, \mu^2, ..m_h^2, ..) = U_l(Q^2, \mu^2, ..m_h^2, ..) + A_l(Q^2/m_h^2, ..) - A_l(\mu^2/m_h^2, ..) \tag{3.16}$$

where all scheme - dependent constants D_l have been canceled. Substituting expression (2.4) and (3.15) into Eq. (3.16) we obtain the more detailed expression, where the gluino contribution is emphasized

$$U_l^{New}(M_Z^2, q_{in}^2, ..m_h^2, ...) = \beta_l \cdot \ln\frac{M_Z^2}{q_{in}^2} - \Delta\beta_l^{\tilde{g}} \cdot \left(\ln\frac{M_Z^2}{m_{\tilde{g}}^2} - I\left(\frac{q_{in}^2}{m_{\tilde{g}}^2}\right) \right)$$

$$-\sum_h \Delta\beta_l^h \cdot \left(\ln(\frac{M_Z^2}{m_h^2}) - \left[\theta(M_h - q_{in})I_l^h\left(\frac{q_{in}^2}{m_h^2}\right) + \theta(q_{in} - M_h)\ln\frac{q_{in}^2}{m_h^2} \right] \right). \tag{3.17}$$

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Here

- at the high energy scale all $I_l^h(M_Z^2/m_h^2)$ are replaced by $\overline{\text{MS}}$'s $\ln(M_Z^2/m_h^2)$ **including** the gluino contribution. This step corresponds to the scheme of $\bar{\alpha}_s(M_Z)$ transformation used by the authors of [2].

- at the low energy scale all $I_l^h(q_{in}^2/m_h^2)$ are turned into $\ln(q_{in}^2/m_h^2)$ in crossing the quark thresholds in $\overline{\text{MS}}$ - scheme, **excepting** the gluino contribution $I_l^{\tilde{g}}$. The latter is because the extracted $\alpha_s(q_{in})$ did not include the gluino contribution at low energy.

The results of the calculations of $\bar{\alpha}_s(M_Z)$ are represented in the last column (MS-MOM-MS) of the Table .

For instructive purposes, let us compare the results of the one-loop evolution of $\bar{\alpha}_s(Q)$ in the standard spline approach Eq.(3.9) and in the three - step procedure (3.13). In the case when q_{in} is located below the lowest threshold, the difference of these expressions is

$$\frac{1}{\alpha(M_Z^2)^{New}} - \frac{1}{\alpha_{\overline{\text{MS}}}(M_Z^2)} = \sum_h \Delta\beta_1^h \left[I_1^h \left(\frac{q_{in}^2}{m_h^2} \right) - \ln \frac{M_h^2}{m_h^2} \right], \qquad (3.18)$$

where the RG solution $\bar{\alpha}_s(Q)^{New}$ correspond to U_l^{New}. In the case when $M_h \gg q_{in}$ and all the particle masses m_i and q_{in} are separated in logarithmic scale, one can evidently obtain $M_h \approx m_h, \quad k \approx 1$ (with some power corrections) for the spline formula (3.9). The same conclusion is evidently valid on the two-loop level. Comparing the expressions (3.18) and (3.11) for the corrections, one can see that there difference includes the constant C_1 only. It is the consequence of the cancelation of the scheme constants in the procedure (3.13).

In the real situation at low energies when $m_1 \sim m_2 ... \sim q_{in}$, *no universal M_n exist in the spline formula.* Moreover, in this case any MS scheme calculations become uncertain owing to the uncertainty in fixing the value of n_f.

4 LEP data and the light gluino window

As is known, the discrepancy between low energy α_s values and the recent LEP data has a chance to be resolved by including the light MSSM gluino $\tilde{g}$ [4],[5],[2].

Our main comment is that in a quantitative discussion of the light gluino mass one should take into account mass effects in the $\bar{\alpha}_s(Q)$ evolution in the ways we propose here. As it follows from our numerical analysis (see three last columns in the Table) the net result for the "gluino existence case" with $m_{\tilde{g}} \simeq m_{c,b}$ effectively shifts $\bar{\alpha}_s(M_Z)$ values corresponding to $m_c < q_{in} < m_b$ by few thousandth which could be physically important.

At the same time we feel it to be premature to discuss in detail the light gluino mass owing to rather big experimental errors of $\bar{\alpha}_s(M_Z)$ "direct" measurements. Nevertheless, consider the final numerical results of the evolution of $\bar{\alpha}_s(Q)$ by the different methods, which are represented in the Table.

In the first column of the Table we have reproduced the $\bar{\alpha}_s(M_Z)$ values following the Bethke recipe (see [1]), i.e. massless evolution with the conjunction at $M_n = m_n$. Note, our first value (0.116) slightly differs from the original one, and for the eighth value we have used the new experimental data [23]. The data on $\bar{\alpha}_s(M_Z)$, the gluino effects included are represented in the next three columns of the Table.

The results of the spline MS evolution of $\bar{\alpha}_s(Q)$, following the massless recipe used by Ellis et al. [2], are presented in the second column. In the next two columns we give a new summary of "measured" values of $\bar{\alpha}_s(M_Z)$ recalculated to scale M_Z using two different smooth mass-dependent procedures including the light $\tilde{g}$: the first is based on the pure MOM scheme, the second – on a three–step procedure (3.13). The application of each of them is dictated by the way of extracting the $\bar{\alpha}_s(Q)$ value in low–energy experiments. One can easily see that the results of both procedures satisfy the following inequality:

$$\bar{\alpha}_s(Q)^{New} \quad \geq \quad \bar{\alpha}_s(Q)^{\overline{MS}} \quad \geq \quad \bar{\alpha}_s(Q)^{MOM}$$

An essential theoretical observation is that the net result of the three–step procedure (3.13) is essentially different from the straightforward one

$$\alpha_s^{\overline{MS}}(q_{in}) \rightarrow \alpha_s^{\overline{MS}}(M_Z) \quad \text{via spline formulae} \tag{4.1}$$

as used in [2] (compare the second and fourth columns in the Table). The difference from the results of evolution in the spline approximation (with $k = 2$, as in [2]) being of the order 5 per cent is important for the discussion of the light gluino hypothesis as it happily works in proper direction raising the "low-energy $\bar{\alpha}_s(M_Z)$ value" (the last column in the Table).

It should be noted, that the growth of the cross–section at the Z–peak (see the tenth line in the Table), obtained in [2], is due to an extra contribution from gluino part proportional to $\Delta\beta_1^{\tilde{g}}$ multiplied by 5/3. At the same time, these authors have used the value $k = 2$ (instead of $exp(5/6)$!) for the threshold–shifting parameter.

The additional growth of $\bar{\alpha}_s(M_Z)$ in the three–step procedure is due to the asymmetry in treating the gluino contribution mentioned above. This growth is of the order $\alpha_s^2 \Delta\beta_1^{\tilde{g}} \cdot 5/3 \approx 0.005$, as it can be seen from the second term in Eq.(3.17).

Note at last that the value of $m_{\tilde{g}}$ (in the sense of the best χ^2 over all data in the Table) reduces to a value less than 0.5 GeV in the first naive procedure and grows up to 10 GeV in the correct second procedure.

5 Conclusion

1. We believe that our analysis is interesting from the theoretical point of view, as it demonstrates the possibility of rather a simple realization of two-loop *mass-dependent* RG calculation of running $\bar{\alpha}_s(Q)$ evolution.

Together with the result of recent publication [24] this opens the door for a systematic consideration of threshold effects in DIS processes.

Our approach could be of importance for the discussion of the "Amaldi et al discrepancy" in GUT scenario. The point is that the massless $\overline{\text{MS}}$ scheme widely used in this case "produces" a systematic shift of the $\bar{\alpha}_s(Q)$ evolution curve with respect to a smooth curve of the mass–dependent case. In the GUT case this shift is not compensated by the "reverse" scheme transition $MOM \to \overline{\text{MS}}$ at $M_{X,Y}$ at the end.

2. Our result seems to be interesting physically as it could contribute into the light gluino discussion. Here, the most annoying data are provided by the first three low energy experimental values in the Table. They are lying systematically below the others data. On the other hand, using these data, one can see the intensification of the "three – step effect" due to the light gluino. To this end, one would compare the columns with and without gluino in the data, we demonstrate below:

N	Bethke-92	MS-MOM-MS	Spline $\overline{\text{MS}}$	MS-MOM-MS
	without gluino		Gluino $m_{\tilde{g}} = 5\text{GeV}$	
1	0.116 ± 0.005	0.1189	0.1309	0.1398
2	0.111 ± 0.006	0.1120	0.1233	0.1289
3	0.113 ± 0.005	0.1135	0.1253	0.1304

However, generally, one should bear in mind that here the launching platform includes experimental data with remarkable small error estimates (that seems to be respected by physical community). The "SuSy race", we are trying to participate in, is extremely fashionable at the moment but nobody is sure about the very existence of a prize. In other words, if further experimental progress will add arguments in favor of the discrepancy, one would be open to discussing other physical mechanisms [25].

Table

Values of $\bar{\alpha}_s(M_Z)$ in the standard QCD, and with MSSM gluino $\tilde{g}$

N	Process	q_{in} GeV	Bethke-92 [1]	Spline $\overline{\text{MS}}$	MOM	MS-MOM-MS
				Gluino $m_{\tilde{g}} = 5$GeV		
1	R_τ [world]	1.78	0.116 ± 0.005	0.131	0.1274	0.1398
2	DIS $[\nu]$	5	0.111 ± 0.006	0.123	0.1204	0.1289
3	DIS $[\mu]$	7.1	0.113 ± 0.005	0.125	0.1220	0.1304
4	$J/\Psi + \Upsilon$ decays	10	0.113 ± 0.007	0.126	0.1220	0.1291
5	$p\bar{p} \to b\bar{b}X$	20	0.109 ± 0.014	0.116	0.1153	0.1210
6	$e^+e^- \to [\sigma_{had}]$	34	0.131 ± 0.012	0.138	0.1376	0.1455
7	$e^+e^- \to$ [ev.shapes]	35	0.119 ± 0.014	0.125	0.1247	0.1310
8	$e^+e^- \to$ [ev.shapes] ([23])	58	0.126 ± 0.007	0.127	0.1262	0.1325
9	$p\bar{p} \to W$ jets	80.6	0.121 ± 0.024	0.1215	0.1210	0.1271
10	$\Gamma(Z^0 \to h)$	91.2	0.130 ± 0.012	0.132 ([2])	0.132	0.132
11	Z^0 [ev. shape]	91.2	0.120 ± 0.006	0.124 ([2])	0.124	0.124
	without π^2 expon. ([2])			or 0.132	or 0.132	or 0.132
	average of $\bar{\alpha}_s(M_Z)$		0.119			0.131

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